

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel International GCSE

Friday 17 November 2023

Morning (Time: 2 hours)

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

P73586A

©2023 Pearson Education Ltd.
Z:1/1/1/




Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1** The equation $kx^2 + 8x + 3k = 0$ where k is a constant, has real unequal roots.

Find the set of values of k giving your answer in an exact simplified form.

(5)

(Total for Question 1 is 5 marks)



2 In triangle ABC , $AB = 3x$ cm, $BC = 5x$ cm and $\angle ABC = 110^\circ$

(a) Find, in degrees to one decimal place, the size of $\angle BCA$

(4)

The area of triangle ABC is 24 cm^2

(b) Find, to 3 significant figures, the value of x

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 2 is 7 marks)

3 A particle P moves along the x -axis.

At time t seconds ($t \geq 0$) the acceleration, $a \text{ m/s}^2$, of P is given by $a = 6t - 16$

When $t = 0$, P is at the origin and is moving with velocity 12 m/s .

(a) Find an expression in terms of t for

(i) the velocity of P at time t seconds

(ii) the displacement of P at time t seconds.

(4)

(b) Hence find the time at which P first returns to the origin.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 3 continued

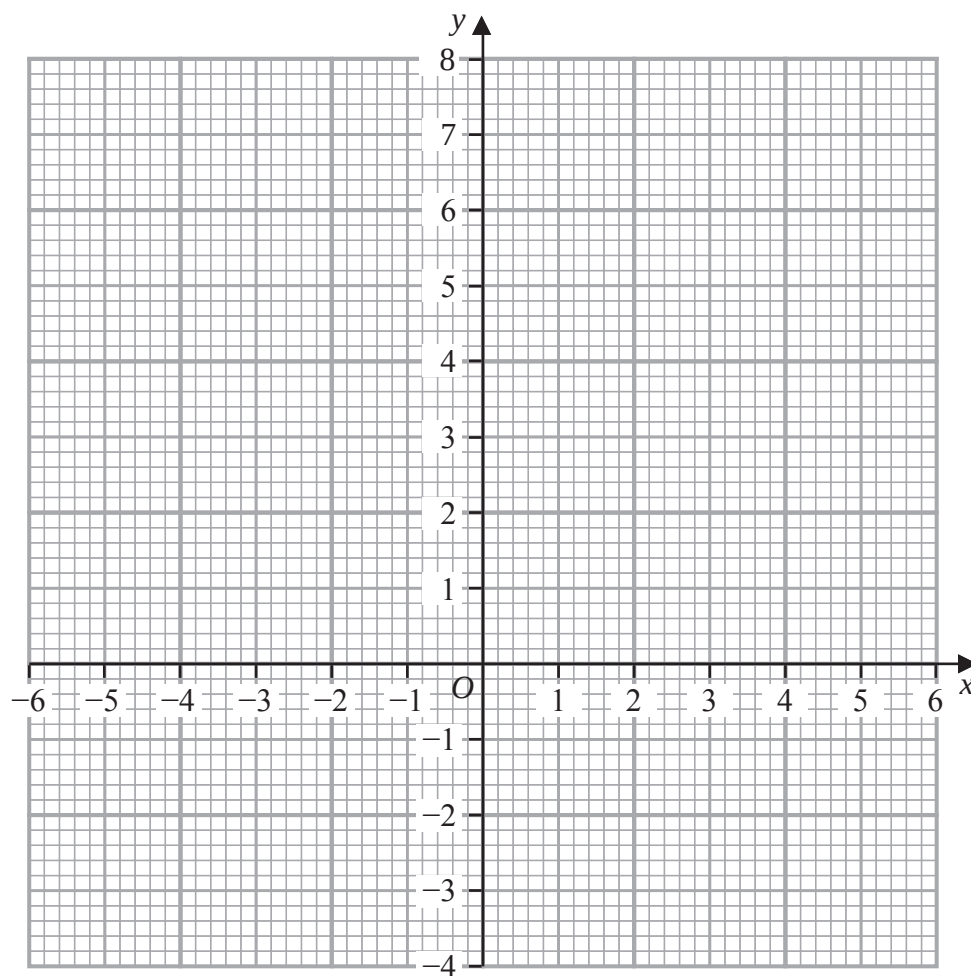
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 3 is 7 marks)

Question 4 continued



Turn over for a spare grid if you need to redraw your graph.



Question 4 continued

DO NOT WRITE IN THIS AREA

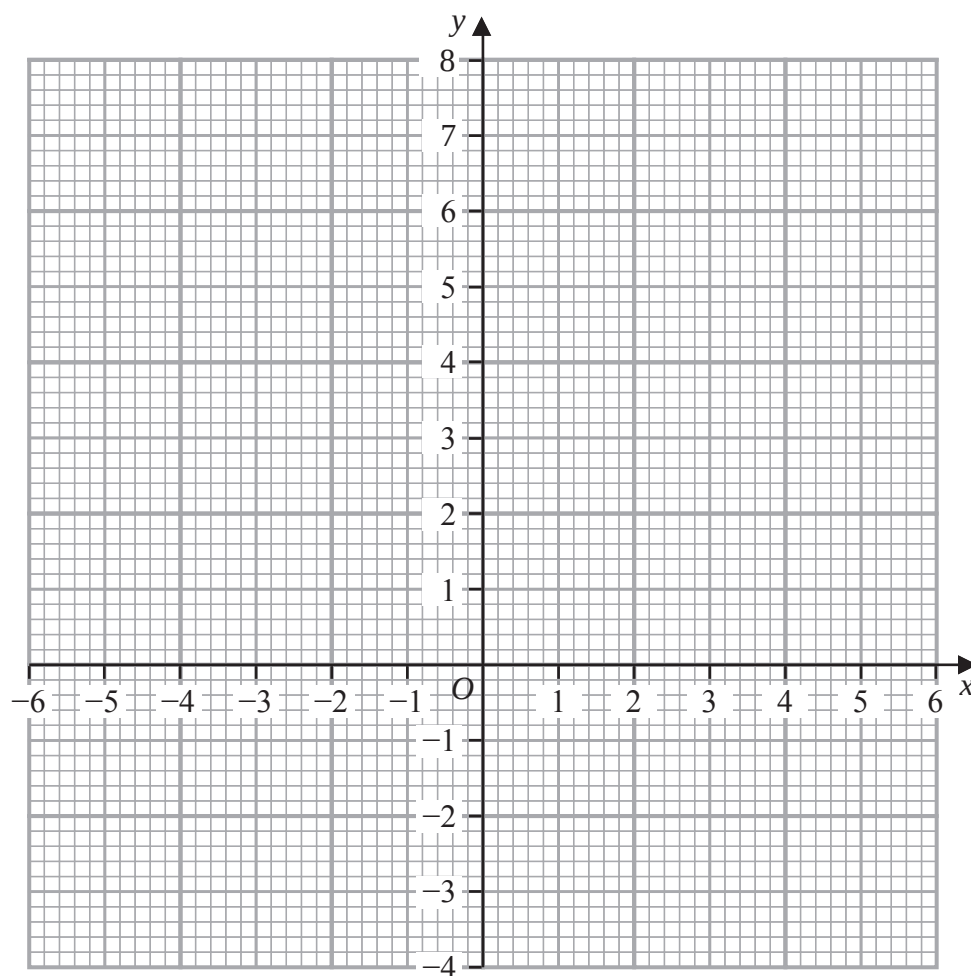
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 4 continued

Only use this grid if you need to redraw your graph.



(Total for Question 4 is 8 marks)



5

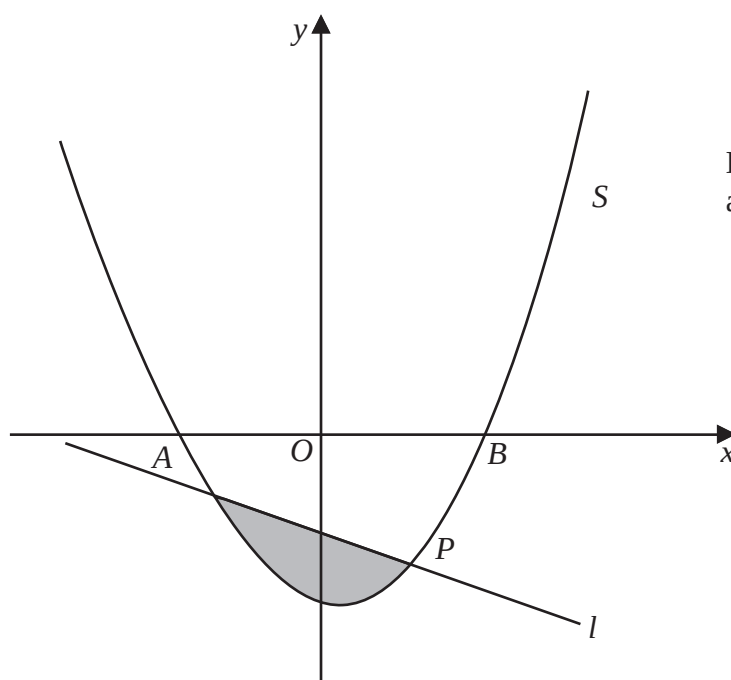


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows part of the curve S with equation $y = px^2 + qx + r$ where p , q and r are constants.

The points A , B and P with coordinates $(-2, 0)$, $(6, 0)$ and $(4, -6)$ respectively lie on S

- (a) Show that an equation of S is $y = \frac{x^2}{2} - 2x - 6$ (3)

The line l is the normal to S at the point P

- (b) Show that an equation of l is $2y + x + 8 = 0$ (5)

The finite region shown shaded in Figure 1 is bounded by S and l

- (c) Use algebraic integration to find the exact area of the shaded region. (7)

.....

.....

.....

.....

.....



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 3 5 8 6 A 0 1 3 3 2

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 5 is 15 marks)



P 7 3 5 8 6 A 0 1 5 3 2

- 6 The volume of oil in a container is $V \text{ cm}^3$ when the height of the oil is $h \text{ cm}$.
Oil is pouring into the container at a constant rate of $12 \text{ cm}^3/\text{s}$.
Given that $V = 3h^3$

find the exact rate, in cm/s , at which the height of the oil is increasing
when $V = 1536 \text{ cm}^3$

(7)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 6 is 7 marks)

P 7 3 5 8 6 A 0 1 7 3 2

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 9 marks)

P 7 3 5 8 6 A 0 1 9 3 2

8 The sum to n terms of an arithmetic series A is S_n

The sum of the first four terms of A is 42 and the fifth term of A is 23

(a) Show that $S_n = \sum_{r=1}^n (Pr - Q)$ where P and Q are prime numbers. (6)

$S_{2n} - 3U_n = 1062$ where U_n is the n th term of A

(b) Find the value of n (4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)

P 7 3 5 8 6 A 0 2 1 3 2

9

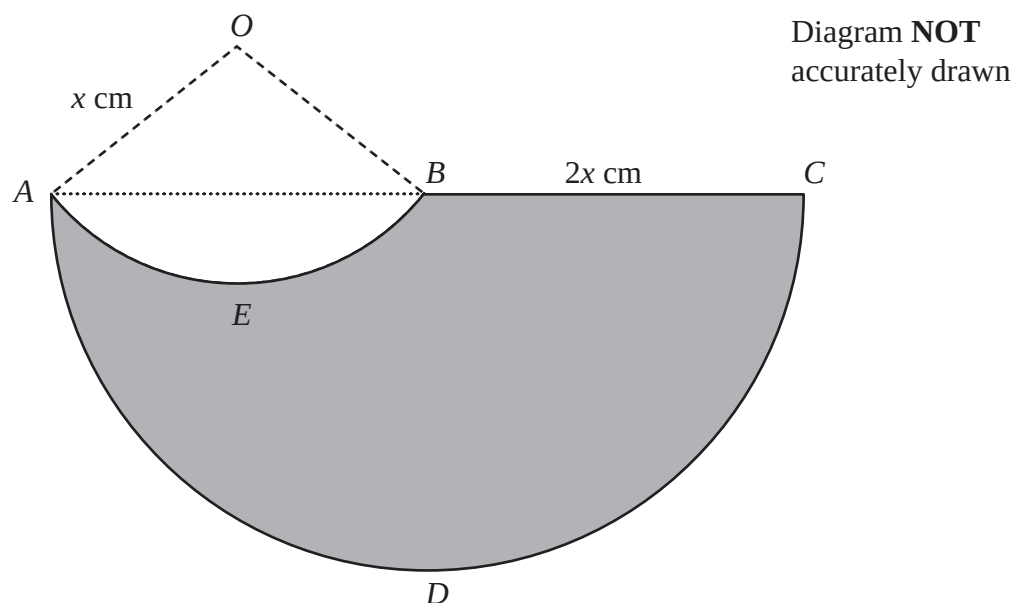


Figure 2

A logo, $AEBCD$, is shown shaded in Figure 2.

The straight line ABC is the diameter of the semicircle ADC

AEB is an arc of a circle with centre O

All angles are measured in radians.

- $BC = 2x$ cm
- $OA = OB = x$ cm
- length of arc $AEB = 1.8x$ cm

The perimeter of the logo is P

(a) Show that $P = ax(\pi + \pi \sin 0.9 + b)$ where a and b are constants to be found.

(7)

Given that $x = 10$ cm,

(b) find, in cm^2 to 3 significant figures, the area of the logo.

(6)

.....

.....

.....

.....



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 3 5 8 6 A 0 2 3 3 2

Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 13 marks)

P 7 3 5 8 6 A 0 2 5 3 2

10 The roots of a quadratic equation are α and β where

$$\alpha + \beta = -\frac{5}{2} \text{ and } \alpha^3 + \beta^3 = \frac{115}{8}$$

(a) Show that $\alpha\beta = 4$

(3)

(b) Form a quadratic equation with integer coefficients, that has roots

$$\frac{\alpha^2 + 1}{\beta} \text{ and } \frac{\beta^2 + 1}{\alpha}$$

(7)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Handwriting practice area with 20 horizontal dotted lines.



P 7 3 5 8 6 A 0 2 7 3 2

Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 10 marks)

P 7 3 5 8 6 A 0 2 9 3 2

11

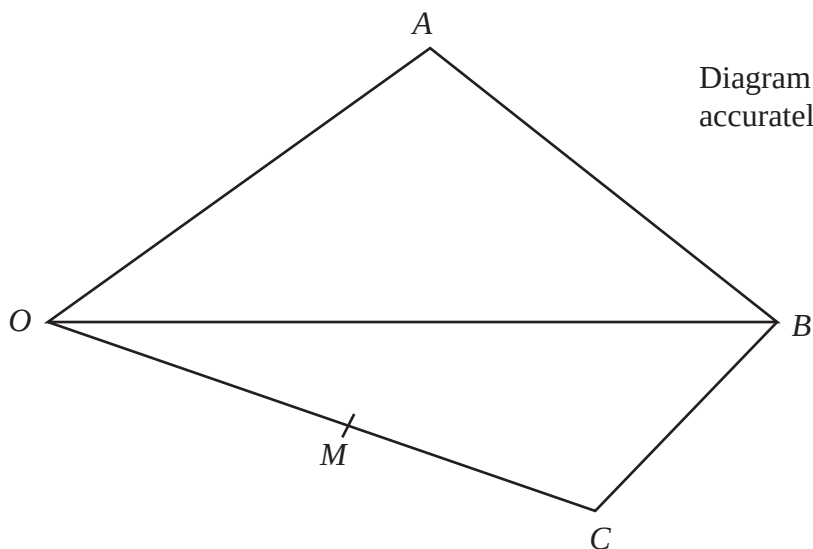


Diagram **NOT**
accurately drawn

Figure 3

Figure 3 shows quadrilateral $OABC$ where

$$\vec{OA} = 4\mathbf{p} + 5\mathbf{q} \quad \vec{OB} = 3\mathbf{p} + \mathbf{q} \quad \vec{OC} = 2\mathbf{p} - 4\mathbf{q}$$

The point M is the midpoint of OC

- (a) Find $\vec{MA}$ as a simplified expression in terms of $\mathbf{p}$ and $\mathbf{q}$

(3)

The point N lies on OB such that M , N and A are collinear.

- (b) Find the ratio $MN : NA$

(6)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



Question 11 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Area for writing the answer to Question 11 continued. The area is ruled with horizontal dotted lines.



P 7 3 5 8 6 A 0 3 1 3 2

Question 11 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS